MODE PROPAGATION THROUGH A STEP DISCONTINUITY IN DIELECTRIC PLANAR WAVEGUIDE

H. Shigesawa, M. Tsuji, and K. Takiyama
Department of Electronics, Doshisha University
Kamikyo-ku, Kyoto 602, Japan

Abstract

This paper proposes two analytical methods which are essential for dealing with the wave propagation through a dielectric step discontinuity; one is a method to accelarate the convergence rate of solutions, especially for TM-mode problems, and the other is a method to treat efficiently the waves with continuous spectrum, in case of open waveguides. It is shown with a number of numerical results that our proposals are quite powerful to the investigation of discontinuity problems.

A. Introduction

Periodically-grooved gratings on a dielectric waveguide are currently being considered for application to integrated circuits in the range from millimeter-wave to optical frequencies. The present authors were successful in analyzing the characteristics of gratings which were partially installed on a planar waveguide[1]. To the dielectric gratings, the step discontinuity is an essential problem to be investigated, and our general interest here is such a case as a surface-wave mode impinges not normally but obliquely to the discontinuity; in addition, the continuous spectrum should be taken into account.

An oblique incidence of a TE or a TM mode produces not only a reflected and a transmitted mode of its own type, but also such modes of the other type in polarization[2]. Therefore, it is essential to a success in solving the problem mentioned above that one can achieve the identical degree in both convergence rate and accuracy of solutions for the incidence of both mode types.

The previous studies which investigated the mode propagation through a dielectric step discontinuity were almost concerned with the cases of TE-mode incidence, and they discussed less extensively the cases of TM-mode incidence. Also, the continuous spectrum was ineffectively discretized to the microwave network approach. Thus, there remains the need for strict investigations on these problems, and that need is satisfied by the present paper.

B. Field Singularity at Dielectric Edges

Let us first consider a step discontinuity in a parallel-plate waveguide that is partially dielectric filled as seen in Fig.1. If the upper conducting plate is removed, we have an open waveguide to be considered later on. There are two dielectric edges along the y axis at $x = t_1$ and t_2 at $z = 0$, which give rise to the singular behavior for the TM-mode incidence and have no longer such an edge effect for the TE-mode incidence. A usual approach, which approximates the field in each guide by the mere truncation of an infinite series of the orthonormal modal functions, solves the boundary problem by means of the mode matching. One thereby misses an important information of the edge effect which is connected with the neglected higher-order modal functions, and suffers a too slow rate of convergence of solutions, especially for TM-mode problems. To recover such an information, Vassallo[3] presented a method based on the application of the Meixner's edge condition[4]. His approach, however, still depends on the full modal expansion of the singular field, and no improvement was achieved in convergence rate.

Our approach recovers the information of the edge effect not in terms of the modal expansion of the singular field, but in terms of straightforward use of the functional form itself, and solves the boundary problem not by a simple mode matching, but by a method fitting the fields in both guides by the sense of least square[5]. To the above end, we assume the x component of the singular electric fields locally bounded around $x = t_p$ at $z = 0$ by $|x - t_p|^{\gamma_p}$, ($p = 1,2$ and $-\frac{1}{2} < \gamma_p < 0$), and approximate the electric field tangential to the discontinuity plane in guide 1, for example, as follows:

$$E_x = \sum_{n=0}^{N} (\delta_{nq} + R_n)\, e_{xn}(x) + \sum_{p=1}^{2} \Gamma_p [\, |x - t_p|^{\gamma_p} - \sum_{n=0}^{N} A_{n,p}\, e_{xn}(x)] , \qquad (1)$$

where the TMq-mode incidence is assumed and R_n, Γ_p are unknown coefficients to be solved, while $A_{n,p}$ are known. It is clear that our approach has only to calculate a few number of amplitudes of (1) for $n \leq N$, unlike Vassallo's approach, and does not encounter the serious difficulty that his approach does, thereby yielding a satisfactory rate of convergence as seen later on.

C. Discretization of Continuous Spectrum

In case of the discontinuity problem in open waveguide mentioned above, one must always consider an appreciable coupling between the discrete surface-wave modes and the waves with continuous spectrum, besides the edge singularity. Owing to the presence of this continuous spectrum, it is customary to discretize it by introducing the Laguerre transform[6],[7]. In a usual discontinuity, however, most of energy carried by an incindent surface-wave mode couples strongly to a part of continuous spectrum in a limited narrow range. To such a case, the Laguerre transform always causes the convergence difficulty because it is effective only for a

1984 IEEE MTT-S DIGEST

so-called good function behaving well over the entire range of the continuous spectrum.

To circumvent such a difficulty, we divide the continuous spectrum into three ranges: one corresponds to the radiation part of it, the second is an optimally scaled extent of the reactive part, and the third, disregarded here, is the rest of the reactive part. To follow this approach, we have only to discretize independently the spectrum in each range. To this end, we employ the Legendre transform to which the normalized Legendre functions provide the complete set of basis functions in each range. On the other hand, the singular fields $E_{sp}(x)$, (p = 1,2) in this case are assumed as follows:

$$E_{sp} = \begin{cases} | x - t_p |^{\gamma_p} , & x < 2t_p \\ t_p^{\gamma_p} \exp[\gamma_p (x - 2t_p) / t_p] , & x > 2t_p \end{cases} \qquad (2)$$

where the decaying $E_{sp}(x)$ beyond $x = 2t_p$ is assumed so as to assure the convergence of integration with respect to x.

D. Numerical Results

We assume first the discontinuity described by the parameters $t_1/t_2 = 1.2$, $d/t_1 = 2.0$, and $k_0 d = 5.0$ in Fig.1, so that the only TM_n modes (n = 1,2) are above cutoff in each guide, and it is considered that the fundamental TM_0 mode is incident normally to the step from the left-hand side of guide 1. We compute the reflected and the transmitted powers of TM_0 and TM_1 modes, the degree of power conservation (total power), and the least mean-square error at the boundary, by considering a number of modes below cutoff. Table 1(a) shows the reasults for different number of modes N. In this calculation, nothing is considered on the edge singularity, and it is found that the usual approach barely ensures the power conservation of 100.000 % at N = 200. On the other hand, Table 1(b) shows the results obtained by our method. We can clearly recognize a remarkable difference in approach; ours easily attains the same degree of power conservation and the mean-square error less than 0.001 % at N = 20; one-tenth in the number N necessary for the usual approach.

Fig.2 summarizes the caluculated mean-square error as a function of different number N. If the upper conducting plate is placed farther above, for example, $k_0 d = 25$, it is necessary to take a huge number N (a few hundreds) to achieve the error less than 0.001 % in usual approach, while ours needs only N ≅ 40 (see also Fig.2). Such a dramatic decrease in N suggests that our approach has a great value in investigating the discontinuity problems.

Let us next consider a step discontinuity in an open dielectric waveguide which is given by that of Fig.1 with the removed upper metal plate. We again analyze the TM_0 mode incidence to the step from the left-hand side. Table 2(a) shows the results obtained without any consideration on the edge condition, while Table 2(b) indicates the results obtained by our approach comprising both the Legendre transform and the consideration on edge singularity in our way. As expected, the present approach improves the magnitudes of total power and the error by a figure or more to those appeared in Table 2(a). Therefore, we may conclude that our approach is quite effective even for the discontinuity problem in open waveguides.

Fig.3 shows the reflection, transmission, and radiation powers as a function of t_2/t_1. The relative transmission power is 100 % at t_2/t_1 = 1.0, as it should, since the discontinuity disappears. As t_2/t_1 decreases, the transmission power goes to zero, while the radiation power reaches almost 100 % and the reflection power goes to its small limiting value, since the surface-wave mode is no longer guided in guide 2 for $t_2/t_1 = 0$.

Fig.4 shows the radiation patterns, where the peak value is normalized to unity for each radiation pattern. Since the TM_0 mode has the Ex component symmetric to the y-z plane at x = 0, radiation occurs into the end-fire (z) direction for $t_2/t_1 = 0$. As t_2/t_1 increases, the angle of radiation peak changes monotonously from zero to a limiting angle of elevation on account of the step discontinuity.

Although additional interesting data will be shown at the presentation, the investigations presented here are sufficient to conclude the purpose of this paper, and are currently applying to the development of practical integrated-circuit components.

Acknowledgement: The authors are indebted to Professor A. A. Oliner of Polytechnic Institute of New York for his constant interest in this work.

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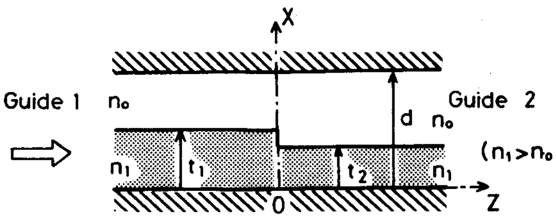

Fig.1. Step Discontinuity in dielectric waveguide .(For an open configuration, the upper conducting plate will be removed ; $t_1/t_2 = 1.2$, $d/t_1 = 2.0$, $k_0 d = 5.0$, $n_1 = 1.46$, $n_0 = 1.0$)

TABLE 1 Characteristic Values Calculated for Different Number N of Expansion Terms, in Case of a Closed Structure. (a) Usual Approach; (b) Present One.

(a)

N	Reflected Power [%]		Transmitted Power [%]		Total Power [%]	Error [%]
	TM_0 mode	TM_1 mode	TM_0 mode	TM_1 mode		
10	0.001	0.010	99.546	0.039	99.595	0.354
20	0.001	0.013	99.796	0.046	99.856	0.102
30	0.001	0.014	99.891	0.048	99.954	0.039
40	0.001	0.014	99.907	0.048	99.972	0.024
50	0.002	0.014	99.918	0.049	99.982	0.016
100	0.002	0.015	99.932	0.049	99.997	0.004
150	0.002	0.015	99.934	0.049	99.999	0.001
200	0.002	0.015	99.934	0.049	100.000	0.000
250	0.002	0.015	99.934	0.049	100.000	0.000

(b)

N	Reflected Power [%]		Transmitted Power [%]		Total Power [%]	Error [%]
	TM_0 mode	TM_1 mode	TM_0 mode	TM_1 mode		
5	0.001	0.015	99.925	0.049	99.990	0.009
10	0.002	0.015	99.929	0.049	99.994	0.004
15	0.002	0.015	99.935	0.050	100.001	0.001
20	0.002	0.015	99.935	0.049	100.000	0.000
25	0.002	0.015	99.934	0.049	100.000	0.000
30	0.002	0.015	99.934	0.049	100.000	0.000

TABLE 2 Characteristic Values Calculated for Different Number N of Expansion Terms, in Case of an Open Structure. (a) Usual Approach; (b) Present One.

(a)

N	Reflected Power (TM_0)	Transmitted Power (TM_0)	Radiation Power		Total Power [%]	Error [%]
			Reflected	Transmitted		
1	0.000	98.384	0.027	0.069	98.481	1.107
2	0.000	98.483	0.024	0.072	98.579	1.019
3	0.000	98.474	0.019	0.062	98.555	0.976
4	0.000	98.807	0.006	0.080	98.889	0.753
5	0.000	99.053	0.010	0.070	99.134	0.659
6	0.001	99.569	0.015	0.070	99.655	0.435
7	0.001	99.531	0.018	0.068	99.617	0.356
8	0.001	99.530	0.019	0.068	99.618	0.329
9	0.001	99.530	0.019	0.068	99.618	0.326

(b)

N	Reflected Power(TM_0)	Transmitted Power(TM_0)	Radiation Power		Total Power [%]	Error [%]
			Reflected	Transmitted		
4	0.001	99.822	0.039	0.075	99.937	0.117
5	0.002	99.972	0.034	0.085	100.093	0.065
6	0.002	99.873	0.028	0.080	99.982	0.054
7	0.001	99.849	0.030	0.081	99.962	0.038
8	0.001	99.838	0.025	0.076	99.940	0.033
9	0.001	99.848	0.027	0.078	99.954	0.032

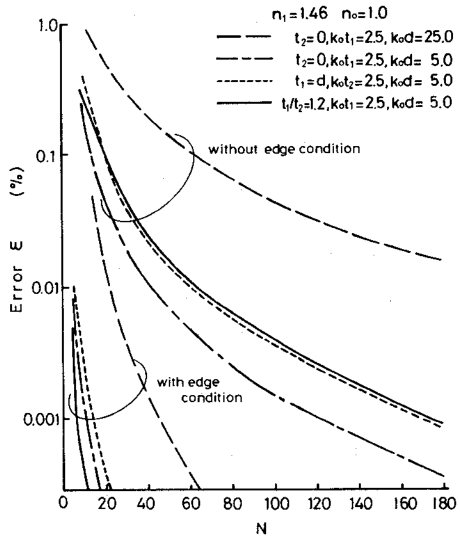

Fig.2. Boundary error ε as a function of the number N of expansion terms. Both cases of $k_0 d$ = 5.0 and 25.0 are shown; $k_0 = 2\pi/\lambda_0$.

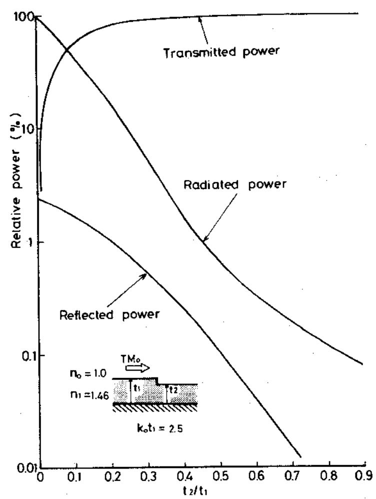

Fig.3. Reflection, transmission, and radiation powers as a function of t_2/t_1, for the TM_0-mode incidence in open structure.

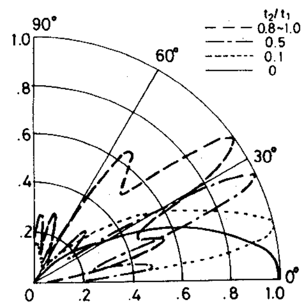

Fig.4. Radiation, patterns in the forward direction for different ratio t_2/t_1.